Of the many design factors involved in an experimental design, we look for two particular types.

Control factors affect primarily the *S/N* ratio, but not the mean. These factors are first set at the appropriate levels so as to minimize the output variability.
Signal factors affect primarily the mean response of the performance characteristic.

The strategy in setting design parameters is to first use the control factors to minimize output variability and then employ the signal factors to move the mean to the desired target. The design parameters that prove to be neither control nor signal factors are set at their low-cost settings, since they do not affect the performance. Fundamental to the Taguchi method is the approach of economical maximization of the output performance characteristic while minimizing the effect of output variability.

12.6 ROBUST DESIGN

Robust design is the systematic approach to finding optimum values of design factors which lead to economical designs with low variability. Taguchi achieves this goal by first performing *parameter design,* and then, if the conditions still are not optimum, by performing *tolerance design.*

Parameter design is the process of identifying the settings of the design parameters or process variables that reduce the sensitivity of the design to sources of variation. In parameter design an accurate modeling of the mean response is not as important as finding the factor levels that optimize robustness. Thus, once the variance has been reduced the mean response should be easily adjusted by using a suitable design parameter, known as the signal factor. An important tenet of robust design is that a design found optimum in laboratory experiments should also be optimum under manufacturing and service conditions. Also, since product designs are often broken down into subsystems for design purposes, it is vital that the robustness of a subsystem not be affected by changes in other subsystems. Therefore, interactions among control factors are highly undesirable.

12.6.1 Parameter Design

Parameter design makes heavy use of statistically planned experiments. Two-and three-level orthogonal arrays are most often used.[1] All common fractional factorial designs are orthogonal arrays. These arrays have the pairwise balancing property that every setting of a design parameter occurs with every setting of all other design parameters the same number of times. They keep this balancing property while minimizing the number of test runs.

1. G. Taguchi, "System of Experimental Design: Engineering Methods to Optimize Quality and Minimize Cost," two vols, Quality Resources, White Plains, NY, 1987; M. S. Phadke, "Quality Engineering Using Robust Design," Prentice-Hall, Englewood Cliffs, NJ, 1989; T. B. Barker, "Engineering Quality by Design: Interpreting the Taguchi Approach," 2d ed., Marcel Dekker, New York, 1994; W. Y. Fowlkes and C. M. Creveling, "Engineering Methods for Robust Product Design," Addison-Wesley, Reading, MA, 1995.

A Taguchi-type parameter design of experiments consists of two parts: (1) a design parameter matrix and (2) a noise matrix (Fig. 12.7). The design parameter matrix specifies the test settings of the design parameters. In Fig. 12.7 there are four parameters $\theta_1, \theta_2, \theta_3, \theta_4$, each tested at three levels for nine test runs in the design parameter matrix. The noise matrix consists of three noise factors, w_1, w_2, w_3, each at two levels. The complete experiment consists of a combination of the design parameter matrix and the noise matrix. Each test run of the design parameter matrix is crossed with all the rows of the noise matrix. Thus, for test run 1, there are four trials, one for each combination of the factors in the noise matrix, like humidity, operator experience, etc. For test run 2, there are another four experiments, etc., so that all told $9 \times 4 = 36$ test conditions will be run. The performance characteristic is evaluated for each of the four trials in the first test run and performance statistics like the mean and the signal-to-noise ratio are determined. This is done for each of the nine trials of the design performance matrix.

The ability to carry out an experimental design of this type will depend on the cost of the experiments and the time required to complete them. If experiments are very expensive to run then it will not be possible to employ a complete outer array and only the noise factor thought to be the most important will be used. In cases where product and process performance can be modeled accurately then an extensive statistical design can be done rather inexpensively. However, experience has found that time and money spent in establishing a robust design or process conditions pays off handsomely in improved quality and reduced costs.

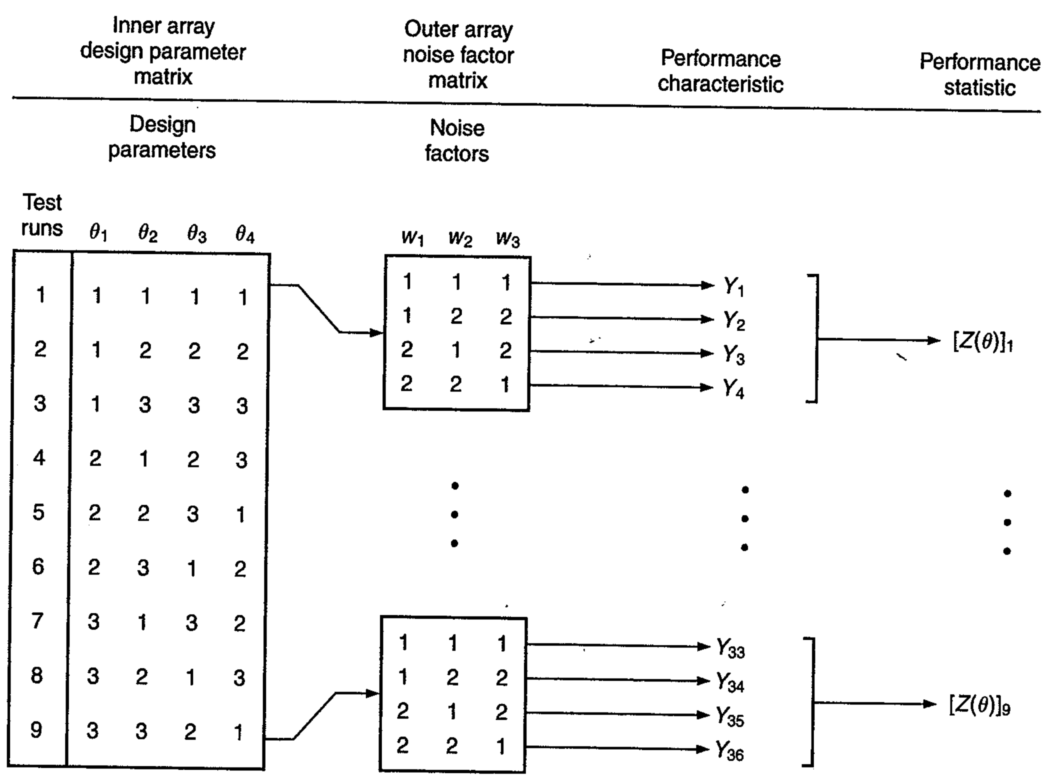

FIGURE 12.7
A typical Taguchi design of experiments for parameter (robust) design. (*After R. N. Kackar, Quality Progress, p. 27, December 1986.*)

EXAMPLE 12.6. The problem is to select the parameters of a compressed-air cooling system (Fig. 12.8) so as to minimize the system cost.[1] The air is cooled first in a precooler, then in a refrigeration unit. Water passes through the condenser of the refrigeration unit, then into the precooler, and finally to the cooling tower, where heat is rejected to the atmosphere. The flow rates of air and water, and critical temperatures are given in Fig. 12.8.

The total cost of the system is the sum of the cost of the refrigeration unit (X_1), the precooler (X_2), and the cooling tower (X_3). Cost equations good for preliminary design have been established.[2]

$$X_1 = 1.20a\,(T_3 - 10)$$

$$X_2 = 1.20b\,\frac{95 - T_3}{T_3 - T_1}$$

$$X_3 = 9.637c\,(T_2 - 24)$$

and the nominal values of the cost parameters are $a = 48$, $b = 50$, and $c = 25$.

Noise factors are factors that cannot be controlled or are too expensive to control. In this problem it was decided that the critical noise factors are:

N_1 = cost parameter for refrigeration unit
N_2 = output temperature of water from cooling tower
N_3 = input temperature of air into precooler

The quality characteristic to be observed is the total cost of the system.

$$C_T = X_1 + X_2 + X_3$$

In optimization terminology, this is the objective function. We wish to find the set of design parameters that minimize C_T subject to the constraints of the mass and energy balances.

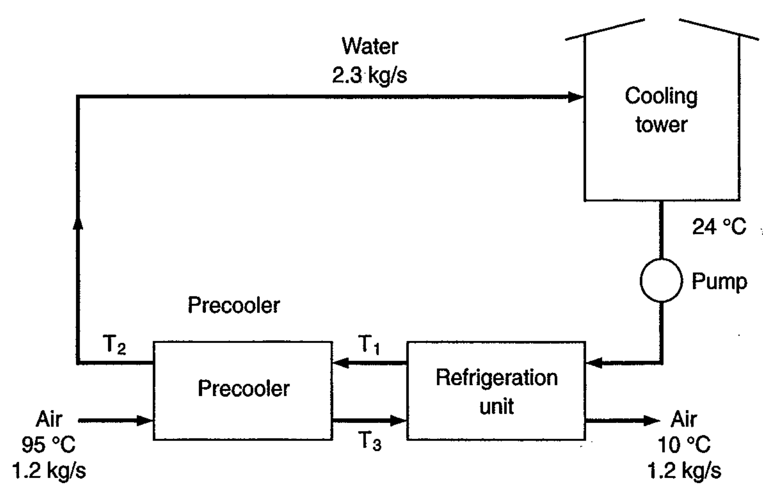

FIGURE 12.8
Air cooling system in Example 12.6

1. R. Umal and E. B. Dean, *Jnl. of Parametrics,* vol. 11, no. 1, 1991. See http://mijuno.larc.nasa.gov for this paper and much more on concurrent engineering methodologies under Design for Competitive Advantage.
2. W. F. Stoecker, "Design of Thermal Systems," 3d ed., McGraw-Hill, New York, 1989, pp. 148–151.

The control factors that can be changed by the designer are the output temperatures T_1, T_2, and T_3. We study these at three levels, low (1), medium (2), and high (3). After a preliminary study it was decided to include three noise factors at two levels. The choice of test conditions for the three control factors and three noise factors is shown below.

Control factor levels	1	2	3
T_1	25	28	31
T_2	36	39	42
T_3	35	38	41

Noise factor levels	1	2
N_1	48	56
N_2	24	27
N_3	95	100

Next we need to select the appropriate orthogonal array for the inner array and the outer array. We need to determine the number of degrees of freedom to find the minimum number of experiments that must be performed. One degree of freedom is associated with the overall mean, regardless of the number of control factors. Next we add the degrees of freedom associated with each control factor, i.e., the number of levels minus one. Therefore, the total number of degrees of freedom is $1 + 3\,(3 - 1) = 7$. We need to conduct at least seven experiments. Taguchi has recorded 18 standard orthogonal arrays.[1] We select the L_9 array (Fig. 12.9*a*), the smallest three-level orthogonal array. The fact that we have only three control factors and the L_9 array is set up for four factors is not a problem. We just leave one of the columns blank. Orthogonality is not lost by keeping one or more columns of an array empty. For the noise array we select an L_4 array (Fig. 12.9*b*).

Expt. number	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

(*a*) $L_9(3^4)$ Orthogonal array

Expt. number	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

(*b*) $L_4(2^3)$ Orthogonal array

FIGURE 12.9
Two examples of orthogonal arrays for use with Taguchi method. (*a*) L_9 experimental design for 4 control factors, each at 3 levels; (*b*) L_4 experimental design for 3 factors, each at 2 levels.

1. G. Taguchi and S. Konishi, "Orthogonal Arrays and Linear Graphs," American Supplier Institute, Dearborn, MI, 1987.

The procedure is as follows. Fig. 12.9*a* shows the combinations of the three control factors, i.e., whether the factor is set at its low value (1), its nominal value (2), or its high value (3) for each of the 9 experiments. Each of these experiments is conducted four times, with the values of the noise factors set at the values designated by the outer array. Thus, $9 \times 4 = 36$ experiments are required, but this is far less than the $4(3^4) = 324$ experiments that would be needed if a statistically designed experiment had not been used.

We now evaluate the quality characteristic, total cost, using the cost equations. These are the responses listed in Table 12.5. Note that column 3 in the L_9 orthogonal array was left empty since we had only three control parameters in this problem.

In traditional statistical analysis we would utilize the means of the responses. In the Taguchi method we use the signal-to-noise ratio *S/N*. Using the *S/N* takes both the mean and variability of the response into account. Because this problem deals with finding the design parameters that minimize cost, we use the smaller-the-better form of *S/N* [Eq. (12.13)]. The *S/N* for the four experiments run for row 1 of the control matrix is

$$S/N = -10 \log \left(\frac{1}{n} \sum y_i^2 \right)$$

$$= -10 \log \left\{ \frac{1}{4} [(4691)^2 + (3998)^2 + (4961)^2 + (4208)^2] \right\}$$

$$= -10 \log (20{,}077{,}067) = -10(7.302700) = -73.03$$

TABLE 12.5
Matrix of experimental responses

Outer array (noise matrix)

	N_1	N_2	N_3
1	48	24	95
2	48	27	100
3	56	24	100
4	56	27	95

Inner array (control matrix)

	T_1	T_2	3	T_3	Responses y_{ij}				Mean	σ
1	25	36		35	4691	3998	4961	4208	4468	441
2	25	39		38	5489	4790	5782	5036	5274	445
3	25	42		41	6325	5621	6641	5899	6122	451
4	28	36		41	4926	4226	5247	4501	4725	452
5	28	39		35	5568	4888	5851	5086	5348	440
6	28	42		38	6291	5598	6590	5838	6079	446
7	31	36		38	4993	4312	5304	4539	4787	447
8	31	39		41	5723	5031	6051	5298	5526	452
9	31	42		35	6677	6029	6991	6194	6473	442

The complete results for the calculation of *S/N* are given below.

Signal-to-noise ratio

	Control matrix			
	T_1	T_2 3	T_3	*S/N* average
1	1	1	1	−73.03
2	1	2	2	−74.47
3	1	3	3	−75.76
4	2	1	3	−73.52
5	2	2	1	−74.59
6	2	3	2	−75.70
7	3	1	2	−73.63
8	3	2	3	−74.87
9	3	3	1	−76.24

A standard approach to analyzing these data would be to use the analysis of variance (ANOVA) to determine which factors are statistically significant (see Sec. 10.11). The Taguchi approach uses a simpler graphical technique to determine which factors are significant. Since the L_9 experimental design is orthogonal, it is possible to separate out the effect of each factor. This is done by looking at the control matrix and calculating the average *S/N* for each factor at each of the three test levels. For example, factor T_3 was at level 2 in experiments 2, 6, and 7. The average *S/N* for this condition is $(-74.47\ -75.70\ -73.63)/3 = -223.80/3 = -74.60$. This calculation is made for each of the three control parameters at each of the three test levels, and the results are tabulated in the response table.

Response table

	Average *S/N*		
Level	T_1	T_2	T_3
1	−74.42	−73.39	−74.62
2	−74.60	−74.64	−74.60
3	−74.91	−75.90	−74.72

The average *S/N* ratios are plotted against test level for each of the three control parameters in Fig. 12.10. We note that T_2 has a stronger effect on *S/N* than the other two control parameters. In fact, T_3 is essentially independent of level of test. Note that the largest value of *S/N* (least negative) is the preferred value. This is true of all forms of *S/N*; use them as a guide to move toward the largest value. As a result of the plots in Fig. 12.10 we conclude that the optimum settings for the control parameters are:

Control parameter	**Optimum level**	**Parameter setting, °C**
T_1	1	25
T_2	1	36
T_3	2	38

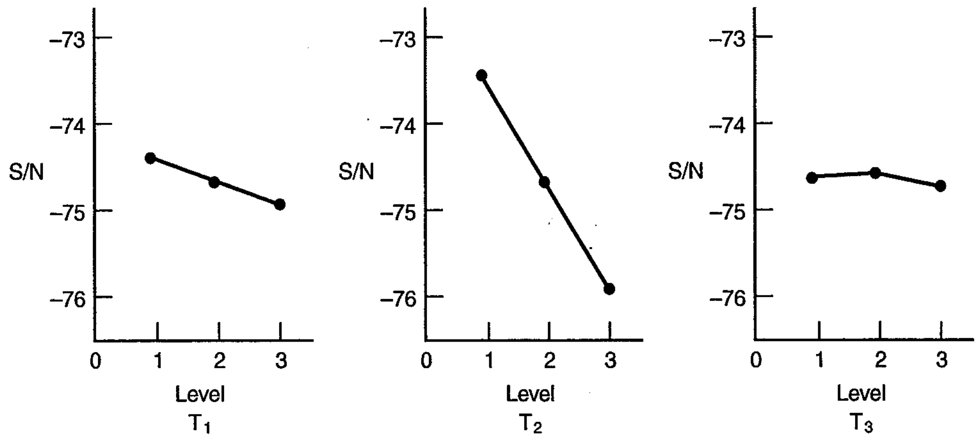

FIGURE 12.10
Linear graphs showing *S/N* for the three control parameters.

Note that none of the experiments carried out in the robust design used exactly this combination of test levels. The initial choice of control parameters was for each to be at level 2. The total cost for this design selection was \$5357, with a standard deviation of 445.6 and a $S/N = -74.60$. At the new optimum for control parameters the cost is \$4551, about a 15 percent reduction. The standard deviation is a bit less, and $S/N = -73.19$.

In this example we have used a relatively small number of experiments to study a number of design variables, three control parameters, and three noise parameters. The methodology yielded a new set of design parameters that are closer to an optimum than the original "informed guess" and which are robust to the noise factors. This example was one in which a closed form solution for the design model was available. In many cases computer models will have to be used to handle the mathematics. However, in many cases no design model exists, and the robust design is achieved by running physical experiments in which a quality characteristic is measured to determine the effect of various settings of the control parameters and noise factors. This is very often the case when trying to optimize a complex mechanism or a production process.[1]

12.6.2 Tolerance Design

Often, as in the example above, the parameter design results in a design optimized for robustness and with a low variability. However, there are situations when the variability is too large and it becomes necessary to reduce tolerances to decrease variability. Typically, analysis of variance (ANOVA) is used to determine the relative contribution of each control parameter so as to identify those factors that are worth considering for tolerance tightening, substituting an improved material, or some other

1. T. B. Barker, *Quality Progress,* pp. 32–42, December 1986; R. T. Fox and D. Lee, *Int. Jnl. of Powder Metallurgy,* vol. 26, no. 3, pp. 233–243, 1990.

means of improving quality at an increased cost. Tolerance design is beyond the scope of this text. An excellent readable source is available.[1]

Taguchi's methods of quality engineering have generated great interest in the United States as several major manufacturing companies have embraced the approach. While the idea of loss function and robust design is new and important, many of the statistical techniques have been in existence for over 50 years. Statisticians point out[2] that less complicated and more efficient methods exist to do what the Taguchi methods accomplish. However, it is important to understand that before Taguchi systematized and extended these ideas into an engineering context they were largely unused by much of industry. We owe Taguchi a debt of gratitude for demonstrating the way to achieve quality robust designs in the most complex of situations.

12.7
OPTIMIZATION METHODS

The example described in the previous section is a search for the best combination of design parameters using a statistically designed set of experiments. Generally there is more than one solution to a design problem, and the first solution is not necessarily the best. Thus, the need for optimization is inherent in the design process. A mathematical theory of optimization has become highly developed and is being applied to design where design functions can be expressed by mathematical equations or with finite-element computer modeling. These optimization methods require considerable depth of knowledge and mathematical skill to select the appropriate optimization technique and work it through to a solution. The growing acceptance of the Taguchi method comes from its applicability to a wide variety of problems with a methodology that is not highly mathematical.

By the term *optimal design* we mean the best of all feasible designs. Optimization is the process of maximizing a desired quantity or minimizing an undesired one. Optimization theory is the body of mathematics that deals with the properties of maxima and minima and how to find maxima and minima numerically. In the typical design optimization situation the designer has created a general configuration for which the numerical values of the independent variables have not been fixed. An objective function[3] that defines the value of the design in terms of the independent variables is established.

$$U = U(x_1, x_2, \cdots, x_n) \tag{12.15}$$

Typical objective functions could be cost, weight, reliability, and producibility or a combination of these. Inevitably, the objective function is subject to certain constraints.

1. C. M. Creveling, "Tolerance Design: A Handbook for Developing Optimal Specifications," Addison-Wesley Longman, Reading, MA, 1997.
2. R. N. Kackar, *Jnl of Quality Tech.*, vol. 17, no. 4, pp. 176–209, 1985.
3. Also called the criterion function or the payoff function.